

4.4 Optimization Higher level learning

day 57 = (3)(19)

Connection: Compare/contrast
with Related Rates

Abstract Situation: Two numbers add to 20.
They are both positive.
What is the maximum product?

$$x=10$$

$$x+y=20$$

constraint

$$x=20-y$$

objective function

$$xy=z$$

$$xy=100$$

$$(20-y)(y)=z$$

$$-2y = -20$$

$$\frac{-2}{-2}$$

$$y=10$$

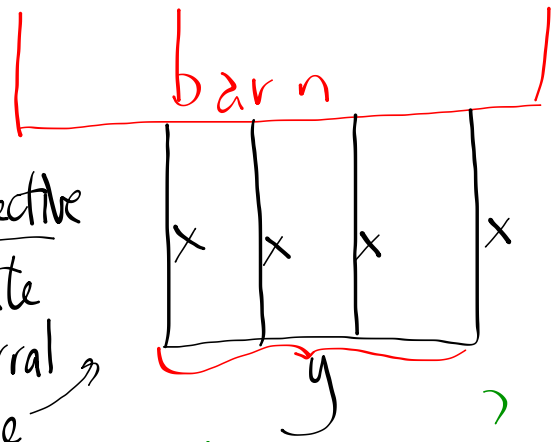
$$\frac{dz}{dy} = \frac{d}{dy}(20y) - \frac{d}{dy}(y^2)$$

$$z' = 20 - 2y = 0$$

"Actual" problem

objective
create a corral like

400 ft of fence



what is the maximum area?

day 57 = (3)(19)

$$4x + y = 400 \quad \downarrow$$

$$A = xy \quad A = (400 - 4x)x$$

$$\frac{dA}{dx} = -4x + (400 - 4x)$$

$$0 = -4x + (400 - 4x)$$

$$4x = 400 - 4x$$

$$8x = 400 \quad x = 50 \text{ ft}$$

$$A = (400 - (4 \times 50)) \times 50$$

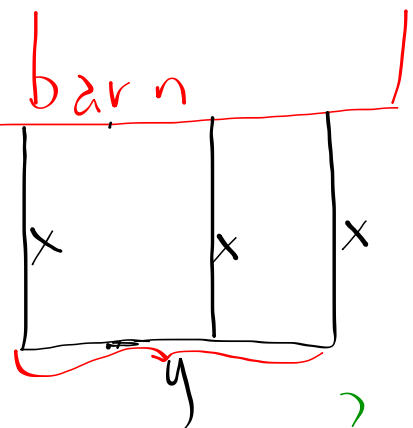
$$A = (200)50$$

$$A = 10,000 \text{ ft}^2$$

"Actual" problem objective
create a corral like

bar n

400 ft of fence



what is the maximum area?

$$3x + y = 400$$

$$y = 3x - 400$$

$$xy = z$$

$$(400 - 3x)(x) = z$$

$$\frac{dz}{dx} = \frac{d}{dx}(400x) - \frac{d}{dx}(3x^2)$$

$$z' = 400 - 6x \quad z' = 0$$

$$3x + y = 400$$

$$3(67) + y = 400$$

$$(201) + y = 400$$

$$y = 199$$

$$x = 67 \quad y = 199$$

$$xy = 13333 \text{ ft}$$

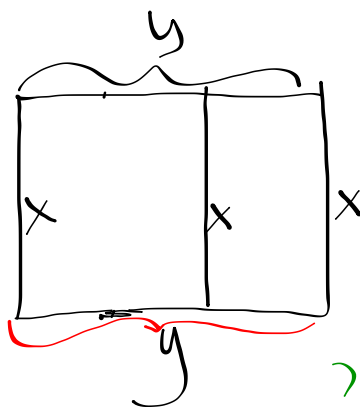
$$400 - 6x = 0$$

$$400 = 6x$$

$$x = 66.66667$$

$$\approx 67$$

day 57 = (3)(19)

"Actual"
problemobjective
create
a corral
like400 ft of
fence

what is the maximum area?

$$2y + 3x = 400$$

$$xy = z$$

$$\frac{2y}{2} = \frac{400 - 3x}{2}$$

$$y = 200 - \frac{3}{2}x$$

$$x(200 - \frac{3}{2}x) = z$$

$$\frac{dz}{dx} = \frac{d}{dx}(200x) - \frac{d}{dx}\left(\frac{3}{2}x^2\right)$$

$$2y + 3\left(66\frac{2}{3}\right) = 400 \quad z' = 200 - 3x$$

$$2y + 200 = 400 \quad 0 = 200 - 3x$$

$$y = 100$$

$$x = 66\frac{2}{3}$$

$$(100)\left(66\frac{2}{3}\right) =$$

$$6,666\frac{2}{3} \text{ ft}^2$$

4.4/1-10 due Monday 11/25

day 57 = $(3)(19)$